**Instructions:** Choose EXACTLY 10 points to be graded. I will evaluate anything you turn in but you MUST indicate which 10 points you want to count towards your grade. You will not earn full credit if your choice is not clear even if all of your solutions are correct. You are free to ask me any questions and work with whomever you like. If you work with any one else or use any resources beyond the textbook and class notes, please let me know. *Quiz 12 in D2L.*

For computations: you do not need to obtain a single value as your answer—it is acceptable to submit an expression that could be plugged into a scientific calculator and give a value. That is, all integrals must be evaluated but not necessarily simplified beyond that.

1. (2 points) Evaluate the indefinite integral $\int x^4 \ln(3x)\,dx$.

2. (2 points) Evaluate the indefinite integral $\int \frac{x}{x^2+3x+2}\,dx$.

3. (2 points) Evaluate the indefinite integral $\int \frac{\cos\left(\sqrt{x}\right)}{\sqrt{x}}\,dx$.

4. (2 points) Evaluate the indefinite integral $\int \arctan(x)\,dx$.

5. (2 points) Evaluate the indefinite integral $\int \frac{1}{x^2\sqrt{4-x^2}}\,dx$.

6. For each improper integral below, determine if it converges or diverges. If it converges, find its value.

   (a) (2 points) $\int_1^\infty x^{-4/3}\,dx$

   (b) (2 points) $\int_2^4 \frac{1}{(x-3)^{3/2}}\,dx$

   (c) (2 points) $\int_1^\infty \frac{\left(\sin(2x)\right)^2}{x^2}\,dx$

7. Let $R$ be the region in the $xy$-plane bounded by $y = x^{2/3}$, $x = 8$, and the $x$-axis.

   (a) (2 points) Compute the volume of the solid obtained from revolving $R$ about the $x$-axis.

   (b) (2 points) Compute the volume of the solid obtained from revolving $R$ about the line $x = 10$.

8. Find the volume of the solid whose base is the region in the first quadrant bounded by the $x$-axis, the $y$-axis, and the graph of $y = 6 - x$, and whose cross-sections perpendicular to the $x$-axis are semicircles.

9. (2 points) Find the arc length of the graph of $y = \ln\left(\sec(x)\right)$ on the interval $[0, \pi/3]$.

10. (3 points) How much work is done in pulling up 15 feet of chain hanging from a 30 foot tower if the chain has a mass of 90 kilograms.

11. For each series below, determine if the series converges absolutely, converges conditionally, or diverges. Justify your answer.

    (a) (2 points) $\sum_{n=0}^{\infty}(-1)^n\left(\frac{5}{4}\right)^n$

    (b) (2 points) $\sum_{n=2}^{\infty}\left(\frac{2}{n-1}\right)^{1.001}$

    (c) (2 points) $\sum_{n=0}^{\infty}\frac{2^n}{n!}$

    (d) (2 points) $\sum_{n=1}^{\infty}\frac{(-3)^n}{n2^{n+1}}$

    (e) (2 points) $\sum_{n=2}^{\infty}\frac{1}{n\ln(n)}$

    (f) (2 points) $\sum_{n=5}^{\infty}2n\cdot 6^{-n^2}$

12. (2 points) For which values of $a$ does the series $\sum_{n=0}^{\infty}\left(\frac{2}{a}\right)^n$ converge?

13. Find the sum.

    (a) (2 points) $1-\frac{2}{3}+\frac{4}{9}-\frac{8}{27}+\cdots$

    (b) (2 points) $\sum_{n=3}^{\infty}\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n-2}}$

    (c) (2 points) $\sum_{n=0}^{\infty}\frac{7\cdot 4^n}{n!}$

    (d) (2 points) $\sum_{n=0}^{\infty}\frac{3(-1)^n}{\pi^{2n+1}(2n+1)!}$

14. For each power series below, determine the radius and interval of convergence of the series.

    (a) (2 points) $\sum_{n=0}^{\infty}(2x)^n$

    (b) (2 points) $\sum_{n=0}^{\infty}\frac{x^n}{n!}$

    (c) (2 points) $\sum_{n=0}^{\infty}\frac{x^n}{n}$

(d) (2 points) $\sum_{n=0}^{\infty} \frac{x}{n!}$

15. (3 points) Use the fourth degree Taylor polynomial for $f(x) = e^{3x^2}$ centered at $x = 0$ to approximate $e^3$. Give an upper bound for the error of your estimate using Taylor's inequality.

16. For each of the functions below, give a power series representation.

    (a) (2 points) $\frac{2}{(1-x)^2}$

    (b) (2 points) $x^2 \ln(1 + x^4)$

    (c) (2 points) $\int \sin(x^3)\, dx$

17. (3 points) Write a power series representation for $\int_0^1 \cos(x^2)\, dx$ then use the Alternating Series Remainder Estimate to approximate this definite integral so that the error is less than $\frac{1}{100}$.

18. Compute the limit.

    (a) (2 points) $\lim_{x\to 0} \frac{2e^x - 2x - 2}{3x^2}$

    (b) (2 points) $\lim_{x\to 0} \frac{\sin(3x)}{x}$

19. (2 points) Solve the differential equation $\frac{dy}{dx} = y^2 \cos(x)$ given that $y(\pi) = 2$.

20. (3 points) Consider the (non-separable) differential equation $\frac{dy}{dx} = x - y$. Suppose $y(0) = 2$. Use Euler's method with four equal subdivisions to approximate the $y$-value when $x = 2$. Show a table including $x$, $y$, $\frac{dy}{dx}$, and the approximation of $\Delta y$.

21. A population of size $P$ increases over time $t$ (in years) at a rate proportional to $P$.

    (a) (1 point) Write a differential equation expressing the relationship above.

    (b) (2 points) Solve the differential equation in part (a) given a population of 100,000 when $t = 0$ and that the population doubles every 7 years. Find the value of $k$ to 5 decimal places.

    (c) (1 point) If $t = 0$ corresponds to January 1st, 2010, give the month and year during which the population will reach 500,000.